verse, which is subtraction or its equivalent, division. But by subtraction we can find only *how many times*; hence, by division the only thing that can be found is *how many times*. That is, how many times one number is contained in another of the same kind. It is thus seen that quotient is always *abstract*. Consequently, the dividend and divisor must be like numbers; for if a quotient is a, the dividend is a times the divisor, whatever it may be. Therefore, a "concrete number" can not be divided by an abstract one.

To find one of the equal parts of a concrete number is more than division: it is a problem that involves the use of division; it is an "application" of division, just as "profit and loss" is an application of percentage. Thus, to find $\frac{1}{4}$ of \$20, we proceed logically as follows:

(*a*) $\frac{1}{4}$ of \$20 is as many dollars as there are 4's in 20.
(*b*) There are *five* 4's in 20.
(*c*) $\therefore$ $\frac{1}{4}$ of \$20 is \$5.

The reasoning in such problems must be in the abstract, and the result interpreted or applied in the conclusion. But pure division involves none of this reasoning—it involves only a retracing of the steps in multiplication or addition.

It is plain that to find $\frac{1}{4}$ of a number is to divide that number by 4. To find $\frac{1}{4}$ of $\frac{8}{9}$ is to divide $\frac{8}{9}$ by 4. Hence, the alleged "compound fraction" is no fraction at all; it is not even an example in multiplication of fractions, as given by all arithmetics, but it is clearly an example in *division* of fractions.

Expressions like $\frac{4}{\frac{2}{3}}$, commonly called "complex fractions", are not fractions; they are indicated *divisions*. They have the *form* of a fraction, but so has an Indian tobacco sign the form of a man. Unexecuted division and ratio may be expressed in fractional form, but a fraction expresses neither division nor ratio. Thus, $8\div9$ may be written $\frac{8}{9}$; but this does not denote 8 of the nine equal parts of "a unit." When $\frac{8}{9}$ expresses a division to be performed, the 9 is a number—*nine;* when it is a fraction, the 9 is a name—*ninths*.

In the former case the expression is read 8 divided by nine; in the latter it is read 8 ninths.

Besides, if $\frac{4}{\frac{2}{3}}$ were a fraction, the denominator $\frac{2}{3}$ would indicate that some unit had been divided into $\frac{2}{3}$ equal parts! Since it is impossible for *man* to so divide a unit, this species of complex fractions must be regarded as a special gift from on high, "for with God all things are possible."

ARITHMETIC.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him

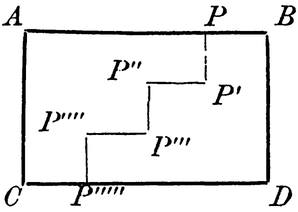
5. **Proposed by E. E. KINNEY, Anaconda, Montana.**

A board is 16 inches long and 9 inches wide. How may it be cut in two

parts that the parts joined together may form a square?

Solution by H. W. DRAUGHON, Clinton, Louisiana.

Let $ABCD$ be the board, and let the broken line $PP'P''P'''P''''$ P''''' be the line of division.



We make $BP=P'P''=P'''P''''=P'''''C=4$ inches, and all parallel to BP; also we make, $PP'=P''P'''=P''''P'''''=3$ inches, and all parallel to BD. From the construction it is obvious that if B be placed at P' and P'''' at C the resulting figure will be 1 foot square.

Solved in a similar mann r by *G. B. M. Zerr*, *Robert J. Aley*, and *J. A. Calderhead.*

PROBLEMS.

12. Proposed by CHARLES E. MYERS, Canton, Ohio.

A man made his will to this effect: that if only the daughter returned home his wife should have $\frac{2}{3}$ and the daughter $\frac{1}{3}$ of the estate; and if only the son returned, his wife should have $\frac{1}{3}$ and the son $\frac{2}{3}$. But the son and daughter both returned. How should the estate be divided?

13. Proposed by J. R. BALDWIN, A. M., Professor of Mathematics in the Davenport Business College, Davenport, Iowa.

A man borrowed \$5000 at a western bank giving his note for \$5000 due in 5 years without grace at 8% interest payable annually, and pays the banker a bonus of \$500 in cash for making the loan; what rate per cent. does he pay?

[Solutions to these problems should be received by April 1st.]

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

PROBLEMS.

9. Proposed by Professor G. B. M. ZERR, A. M., Principal of Schools, Staunton, Virginia.

$$\left.\begin{array}{l} x+y^2+z^2=21 \\ x^2+y^3+z=45 \\ x^3+y+z^2=71 \end{array}\right\}. \text{ Find } x, y, \text{ and } z.$$

10. Proposed by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania,

$$x^2+y^2+w^2+z^2=65\ldots.(1),$$
$$(x+z)^2+(y+w)^2=113\ldots.(2),$$
$$(y+z)^2+(x+y)^2=117\ldots.(3),$$
$$(x+y)^2+(z+w)^2=125\ldots.(4).$$

How many values has each of the four unknown quantities?

11. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

Two men, A and B, had a money-box, containing \$210, from which each drew